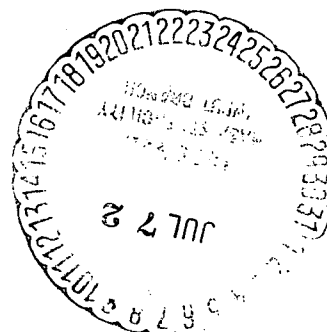


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CALCULATION OF THE PROPERTIES OF RESONATOR
TYPE MUFFLERS

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ABSTRACT. In the preceding paper (this Journal 25 (3) 1969, p. 122) it was shown that the noise reducing effect of a muffler is deduced by means of the four-terminal constants of the equivalent electrical network of the whole muffler system. For expansion chamber type mufflers, the approximate equations are very simple and the noise reducing curve is easily obtained by a graphical method.

In this paper, a similar method is applied to the resonator type mufflers, and equations are shown in Equations (11), (12), (13) and (16), and a graphical method in Table 1 and Figure 8.

Practically, as shown in Figure 7(a), the resonator type muffler is efficient only in the region near the first resonant frequency, and furthermore the loss at the mouth of the resonator is not negligible. But the calculation of the loss is very difficult, so the loss is estimated as convenient to the graphical process as shown in Figure 9.

By this estimation, the noise reducing curve is very easily obtained by a graphical method, and as shown in Figure 7 (b), the resulting curve is useful for practical purposes.

When the connecting points of the cavity of a resonator and the pipe are in two locations or are distributed as in Figure 11, the four-terminal constants are shown in Equations (28) and (31). In a practical case,

when the distance between the two connecting points or the distributing length of connecting points and the total connecting area are very large, the muffler is approximately equivalent to an expansion chamber type muffler, as illustrated in Figure 13.

1. Introduction

/363*

In a previous report [1], it was shown that the effect of a muffler can be expressed by means of the four-terminal constants of the equivalent electrical circuit of the entire muffler system. Simple approximate equations can be obtained for expansion chamber type mufflers, and their properties can be found easily by construction on graph paper.

In this paper, the same concept is applied to resonator type mufflers. Besides establishing the equations, consideration is devoted to practical methods of finding the properties graphically.

First, the paper describes a method of seeking the equation when the loss is ignored and of seeking the properties graphically.

/364

However, in the case of a resonator consisting of a cavity and a neck part, the loss in the neck part cannot be ignored, and the analysis is difficult. Such a procedure cannot necessarily be called simple. Even if the loss in the neck part is taken into consideration, its value cannot be found easily, and it was decided, therefore, to estimate it in a manner convenient for plotting the graphs. In this way, a simple

*Numbers in the margin indicate pagination in the original foreign text.

and practical plotting method was found with fairly good agreement with the measured values.

Furthermore, a theoretical equation was given for cases in which the cavity is connected to the pipe in two places or in which the connections are distributed. Studies were made of these cases.

2. General Equations for Resonator Type Mufflers

When the input pipe and tail pipe of a muffler have cross sections which are small enough in comparison to the wavelength, the effect of such a muffler can be expressed as follows [1]:

$$L = L_s - L_s' + 10 \log R_2'/R_2 \quad (\text{dB}) \quad (1)$$

The terms here with dashes represent the values before the installation of the muffler, and R_2 represents the acoustic radiation resistance at the tail pipe outlet. L_s is expressed as follows by means of the four-terminal constants (ABCD) of the equivalent electrical circuit of the whole muffler system, including the reactance inside the sound source and the radiation reactance at the tail pipe outlet.

$$L_s = 20 \log \left| \frac{B}{Z_1} \right|, \quad L_s' = 20 \log \left| \frac{B'}{Z_1} \right| \quad \left(\begin{array}{l} \text{constant pressure} \\ \text{sound source} \end{array} \right) \quad (2)$$

$$L_s = 20 \log |D|, \quad L_s' = 20 \log |D'| \quad \left(\begin{array}{l} \text{constant velocity} \\ \text{sound source} \end{array} \right) \quad (3)$$

Z_1 is the characteristic impedance of the muffler inlet pipe.

In resonator type mufflers, the resonator is usually installed midway along the pipe, and the equivalent circuit can

be represented in the manner shown in Figure 1(a). S_1 is the cross sectional area of the pipe, and l_1 is the length of the pipe from the resonator to the sound source — that is, the length of the input pipe. l_3 is the length of the pipe after the resonator, that is, the length of the tail pipe. Both lengths are effective lengths after pipe-end corrections have been made. The characteristic impedance of the pipe is $Z_1 = \rho c / S_1$ (ρ is the density of the medium, and c is the acoustic velocity). F_r represents the four-constant matrix of the resonator part.

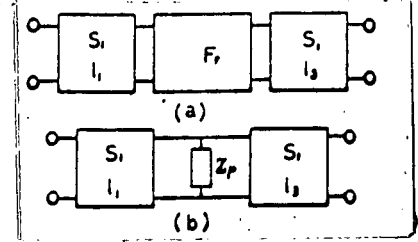


Figure 1. The equivalent network of resonator type mufflers

If we represent F_r as

$$F_r = \begin{bmatrix} A_r & B_r \\ C_r & D_r \end{bmatrix} = \frac{1}{E'} \begin{bmatrix} A' & jZ_1 B' \\ j\frac{1}{Z_1} C' & D' \end{bmatrix} \quad (4)$$

then ($j = \sqrt{-1}$), and if we ignore the loss in the pipe part of the four terminal constants of the entire muffler system, B and D will be:

$$B = j\frac{Z_1}{E'} (A' \cos kl_1 \sin kl_2 + D' \sin kl_1 \cos kl_2 + B' \cos kl_1 \cos kl_2 - C' \sin kl_1 \sin kl_2) \quad (5)$$

$$D = \frac{1}{E'} (-A' \sin kl_1 \sin kl_2 + D' \cos kl_1 \cos kl_2 - B' \sin kl_1 \cos kl_2 - C' \cos kl_1 \sin kl_2) \quad (6)$$

Here, k is the wavelength constant. In other words, generally speaking, L_s will have the following form:

$$L_r = 20 \log \left| \frac{1}{E'} \left\{ A' \begin{matrix} \cos \\ -\sin \end{matrix} kl_1 \sin kl_2 \right. \right. \\ \left. \left. + D' \begin{matrix} \sin \\ \cos \end{matrix} kl_1 \cos kl_2 + B' \begin{matrix} \cos \\ -\sin \end{matrix} kl_1 \cos kl_2 \right. \right. \\ \left. \left. - C' \begin{matrix} \sin \\ \cos \end{matrix} kl_1 \sin kl_2 \right\} \right| \quad (7)$$

The symbols sin and cos used here at the top and bottom have the following meaning. When a constant pressure sound source is used, the upper one is used; when a constant velocity sound source is used, the lower one is used. The same applies also to the equations appearing below.

When the size of the cross section of the part where the resonator is connected to the pipe (area S_p) is sufficiently small in comparison with the wavelength, and if the impedance of the resonator as viewed from the point of connection is Z_p , F_r will be given by

$$F_r = \begin{bmatrix} 1 & 0 \\ 1/Z_p & 1 \end{bmatrix} \quad (8)$$

and the equivalent circuit can be written as in Figure 1 (b).

If the loss is ignored, Z_p will be a pure reactance. If it is assumed that $Z_p = \rho c / S_p$, it will generally be possible to represent it in the following form:

$$Z_p = jZ_s X = jZ_s \frac{S_1}{S_p} X \quad (9)$$

That is, in Equation (4),

$$A' = D' = E' = 1, \quad B' = 0, \quad C' = -\frac{S_p}{S_1} \frac{1}{X} \quad (10)$$

Consequently, Equation (7) will be:

$$L_r = 20 \log \left| \frac{\sin k(l_1 + l_2)}{\cos k(l_1 + l_2)} + \frac{S_p \begin{matrix} \sin \\ \cos \end{matrix} kl_1 \sin kl_2}{S_1 X} \right| \quad (11)$$

3. Simple Resonator Type Mufflers

3.1. Equations When the Loss is Ignored [2, 4]

Figure 2 gives examples of muffler in which simple resonators are installed.

S_r is the cross sectional area of the cylindrical cavity, and l_r is its length. If it is assumed that the sound waves are plane waves advancing in the axial direction and further that kl_p is small, in Figure 2 (a) we have

$$Z_r = jZ, X = jZ_r \left(kl_r - \frac{S_r}{S_p} \frac{1}{\tan kl_r} \right) \quad (12)$$

In Figure 2 (b) and (c), ($l_a + l_b = l_r$)

$$Z_r = jZ, X = jZ_r \left(kl_r - \frac{S_r \cos kl_a \cos kl_b}{S_p \sin kl_r} \right) \quad (13)$$

If we consider only the low frequency range, the resonator can be regarded as a concentrated circuit, and if the volume of the cavity is V_r , both of the above equations will be:

$$Z_r = jZ, X = jZ_r \left(kl_r - \frac{S_r l_r}{V_r} \frac{1}{kl_r} \right) \quad (14)$$

and the resonance will occur at

$$kl_r = \sqrt{\frac{S_r l_r}{V_r}} \quad (15)$$

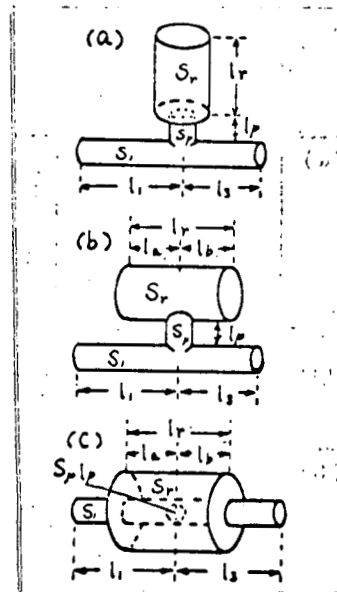


Figure 2. Resonator type mufflers

The muffler shown in Figure 3 is called a side branch type muffler. It can be assumed that this is a

muffler in which the neck part and the cavity in Figure 2 (a) have been combined. Here,

$$Z_p = jZ, X = jZ, \frac{-1}{\tan kl_p} \quad (16)$$

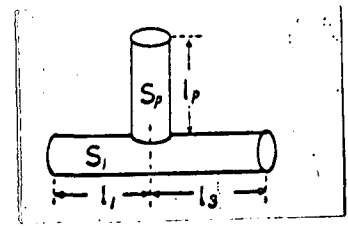


Figure 3. A side branch type muffler

L_S can be calculated by substituting X above in Equation (11), and the effect of installing the muffler can be sought from this and from the L_S before the installation of the muffler.

If a resonator has been installed in a pipe measuring $l_1 + l_3$, we will obtain:

$$L = 20 \log \left| 1 + \frac{S_p \frac{\sin kl_1 \sin kl_3}{\cos k(l_1 + l_3)}}{S_1 X \frac{\sin k(l_1 + l_3)}{\cos k(l_1 + l_3)}} \right| \quad (17)$$

It is clear from the above equation that L_S and L will become large when X has become small, that is, when resonance has occurred. It is also clear that the effect will be greater, the greater is S_p/S_1 , and the smaller is the resonance point [Equation (15)]. In other words, it can be said that in a resonator type muffler, the cross section midway along the pipe is short-circuited by the impedance of the resonator, and the transmission of the acoustic noise power to the tail pipe is inhibited.

3.2. Comparison with Experimental Values

If the size of the tail pipe outlet is small enough in comparison with the wavelength, it will be possible to approximate the radiation resistance (acoustic resistance) at the outlet when there is a point sound source. This is expressed as follows:

$$R_2 = \frac{\omega^2 \rho}{4\pi c} \quad (\omega = 2\pi f) \quad (18)$$

It is also assumed that the sound waves spread out in spherical wave patterns centering around the object.

If the sound pressure at the input pipe inlet is P_1 , the volume velocity at the tail pipe outlet is U_2 , and the sound pressure at a distance r from the outlet is P_s , then the radiated acoustic power will be: /366

$$W_2 = |U_2|^2 R_2 \quad (19)$$

Since R_2 is small,

$$P_1 \approx B U_2 \quad (20)$$

From these, the following is derived:

$$|P_s|^2 = \rho c \frac{W_2}{4\pi r^2} = |P_1|^2 \left| \frac{Z_1}{B} \right|^2 \left(f \frac{S_1}{2cr} \right)^2 \quad (21)$$

$$\therefore 20 \log \left| \frac{B}{Z_1} \right| = 20 \log |P_1| - 20 \log |P_s| - \left(20 \log \frac{2cr}{S_1} - 20 \log f \right) \quad (21)$$

In other words, if the system is built up as shown in Figure 4, and the differences in the level records between P_1 and P_s are determined and corrections to the third term are made, it will be possible to find L_s for a constant sound pressure sound source [2, 4].

Figure 5 gives the results of measurements for a pipe with an effective length of $l_1 = 48.65$ cm in comparison with the theoretical value ($20 \log |B/Z_1| = 20 \log |\sin kl_1|$). It is evident that this measuring method may be used within a range of frequencies which are not too high.

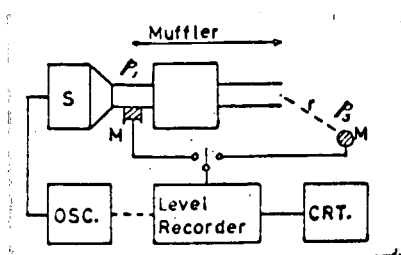


Figure 4. Block diagram of measuring system of L_s (constant pressure source)

S — loudspeaker; M — microphone

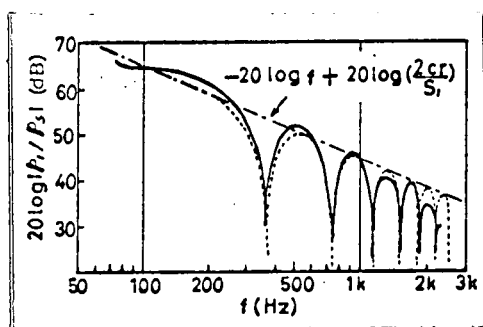


Figure 5. $20 \log |p_1/p_S|$ of a pipe (constant pressure source). The solid curve shows the result of measuring and the dotted curve shows that calculated by Equation (22). $l_1 = 486.5$, $S_1 : 27 \phi$, $r = 100 \sqrt{2}$, (mm), $c = 350$ m/s (30°C).

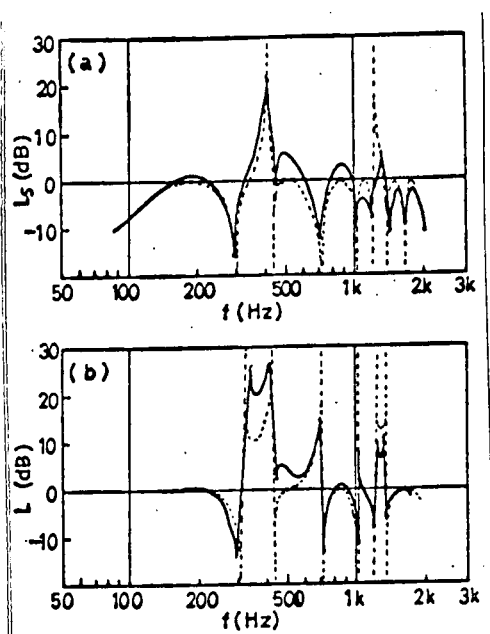


Figure 6. L_s and L of a side branch type muffler. The solid curves show the results of measuring and the dotted curves show those by graphical method.

$l_1 = 400$, $l_3 = 165$, $S_1 : 27 \phi$, $l_p = 220$, $S_p : 16 \phi$ and $l' = l_1 + l_3 = 565$ (mm)

Figure 6(a) shows the measurement results using this measurement method for a pipe measuring $l' = l_1 + l_3$ with a side branch; the results are compared with the results calculated by means of Equation (16) and Equation (11). There is a good correspondence, except that in the calculations the loss was disregarded, so that infinitely large results were obtained. In the measurements, there is some loss, which gives finite results.

In Figure 6 (b) the effect from installation of the side branch L was compared for the measured [1] and calculated values. In this case, the value is greater because of no resonance which was present when there was only a pipe of length L' .

Figure 7 (a) is an example in which a resonator of the type shown in Figure 2 (c) was used. The connecting part consists of 12 holes with a radius of $a_p = 0.5$ cm; L_p is the actual length, 0.3 cm, to which $0.85 a_p$ was added as a compensation length. The valleys marked by x in the figures are caused by the higher harmonics of the sound source speaker (a 20 cm Type MRB speaker), while f_1 and f_2 indicate the resonance frequencies of the resonator. There is a good correspondence within the range where the frequencies are not too high with the following exception: since the loss in the connecting parts was disregarded in the calculations, infinitely large values were found at the resonance points of f_1 , f_2 , etc., but these were finite in the measurement results. It is also evident that the effect is pronounced in the vicinity of the frequencies centering around the first resonance frequency f_1 . In other words, in the calculations, one may approximately assume the resonator to be a concentrated circuit.

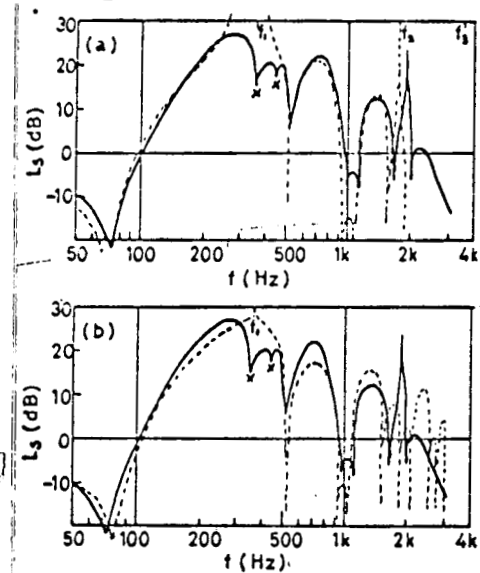


Figure 7. L_s of a muffler illustrated in Figure 2(c) (constant pressure source). The solid curves show the results of measuring and the dotted curves show those by graphical method, (a) by Equation (11) and (13), (b): approximation. The dips marked by x are caused by the harmonics of the source, and f_1 , f_2 and f_3 show the resonant frequencies. $L_a = L_b = 100$, $L_r = L_a + L_b = 200$, $S_r: 135\phi - 33\phi$, $L_p = 7$. $S_p: 10\phi \times 12$, $L_1 = 175$, $L_3 = 306$, $S_1: 27\phi$ (mm)

1367

In a resonator type muffler, the loss in the connecting parts or the neck parts of the resonator cannot be ignored. If the loss resistance at these parts is R_p , the impedance of the resonator is

$$Z_r = R_p + jZ_r X \quad (23)$$

In this case, the impedance at the resonance frequency will be R_p , and not zero, and the effect in the vicinity of the resonance frequency will decrease.

3.3. Finding the Properties by Construction on Graph Paper [2,3]

The curve of the properties can be found easily with the above equations by construction on graph paper, even without performing the numerical calculations.

TABLE 1. RELATIONS BETWEEN
 $G = 20 \log |g_1 + g_2|$ to $G_1 = 20 \log |g_1|$ and $G_2 = 20 \log |g_2|$.

$G_1 = 20 \log g_1 $ and $G_2 = 20 \log g_2 $ (dB)	$G = 20 \log g_1 + g_2 $ (dB)	
	$g_1 \times g_2 < 0$	$g_1 \times g_2 > 0$
$G_1 \gg G_2$	G_1	G_1
$G_1 = G_2 + 20$	$G_1 - 0.9$	$G_1 + 0.8$
$G_1 = G_2 + 10$	$G_1 - 3.3$	$G_1 + 2.4$
$G_1 = G_2 + 6$	G_2	$G_1 + 3.5$
$G_1 = G_2 + 3$	$G_2 - 7.7$	$G_1 + 4.6$
$G_1 = G_2$	$-\infty$	$G_1 + 6.0$

That is, X , as well as L_s or L have the following form:

$$G = 20 \log |g_1 + g_2| \quad (24)$$

If we assume that

$$G_1 = 20 \log |g_1|, \quad G_2 = 20 \log |g_2| \text{ (dB)} \quad (25)$$

the relationship between G , G_1 , and G_2 will be as shown in Table 1 in cases where $G_1 \geq G_2$. Utilizing this, G can be constructed on a graph, as shown

in Figure 8, from the relationship of the signs of g_1 and g_2 and also from the values of G_1 and G_2 [2].

If one prepares graph paper with a logarithmic scale on the abscissa and with a dB scale on the ordinate, curves expressing the triangular functions in terms of dB (curves of

$20 \log |\sin x|$, $20 \log |\cos x|$, and $20 \log |\tan x|$ can be drawn. Then it will be sufficient to slide it over to the left until the point $x = 1$ coincides with the frequency where $k\lambda = 1$. Furthermore, since the sum and the difference of the logarithms can be easily determined on the graph paper [1], the characteristic curves of L_s and L can be found by performing these operations in sequence.

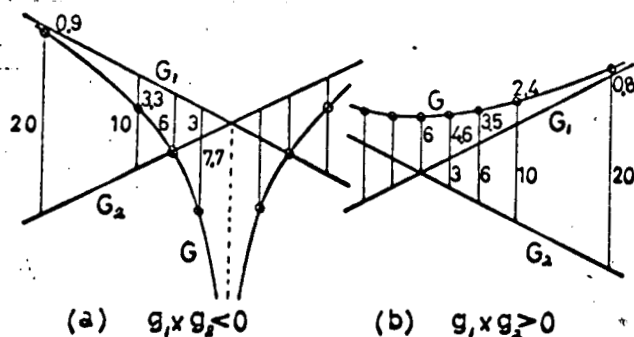


Figure 8. Construction of $G = 20 \log |g_1 + g_2|$ from $G_1 = 20 \log |g_1|$ and $G_2 = 20 \log |g_2|$. Numerals are in dB (cf. Table 1)

The calculated values given in Figures 6 and 7 were found in this manner.

3.4. Finding the Approximate Properties with Assumed Losses

As was mentioned above, with a resonator it is sufficient to consider approximately only the range of concentrated constants. If the loss is ignored, the impedance will be expressed by Equation (14) without relation to the shape of the cavity. For the sake of comparison, $20 \log |X|$ was calculated by Equations (13) and (14) for the cavity used in Figure 7; the results are shown in Figure 9.

Since losses are present in actual fact, the depth of the valley of $20 \log |X|$ becomes shallow in the vicinity of resonance frequency f_1 . However, since it is difficult to calculate

this loss, for the sake of convenience in plotting the graph, let us approximate it roughly by means of two straight lines for $20 \log k l_p$ and $20 \log (S_p l_p / V_r k l_p)$, as in lines (3) and (4) in Figure 9.

In order to plot $20 \log |(S_p / S_1) / X|$ in Equation (11), it is sufficient to draw straight lines for 6 dB/octave and -6 dB/octave, as in Figure 10, from the following points:

$$f_1 = \frac{c}{2\pi} \sqrt{\frac{S_r}{V l_r}} \quad (\text{Hz}) \quad (26)$$

$$20 \log \frac{S_r}{S_1} + 10 \log \frac{V_r}{S_1 l_r} \quad (\text{dB}) \quad (27)$$

Taking the fact into consideration that $X < 0$ when $f < f_1$ and that $X > 0$ when $f > f_1$, it is possible to find the L_S curve easily by making constructions on the graph paper in the proper sequence by the method described above.

Figure 7 (b) compares the curves found in this manner with the measured values. There is a fairly good correspondence.

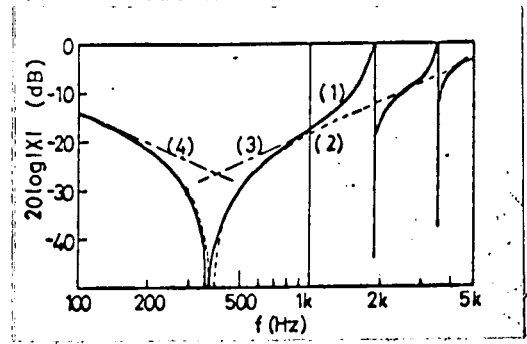


Figure 9. $20 \log |X|$ of a resonator illustrated in Figure 2 (c) and its approximation. Curve (1): Equation (13), curve (2): Equation (14), curve (3): $20 \log k l_p$, and curve (4): $20 \log (S_p l_p / V_r k l_p)$. S_r : 135, ϕ : -33, $l_a = l_b = 100$, $l_r = l_a + l_b = 200$, S_p : 10 $\phi \times 12$, $l_p = 7$. (mm)

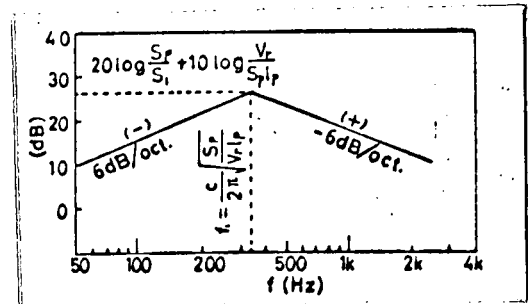


Figure 10. Approximate curve of $20 \log |S_p / S_1 X|$

4. When There are Two or More Connection Points

4.1. Theoretical Equations [2, 5]

Figure 11 shows examples in which the cavity is connected to the pipe at two places or more.

Figure 11(a) is an example in which there are two connections. If it is assumed that the sound waves in the pipe-shaped section are plane waves proceeding in the axial direction, the equivalent circuit for the part corresponding to the resonator will be as shown in Figure 12, and the four terminal parameter F_r can be found by seeking the four terminal constants for the l_2 part of the pipe, the l_a part of the cavity, the l_2 part, the l_c part, and the connecting hole parts in Figure 11 (a), and by constructing these. If it is assumed that kl_p is small, the terms of F_r in Equation (4) will be the following: }

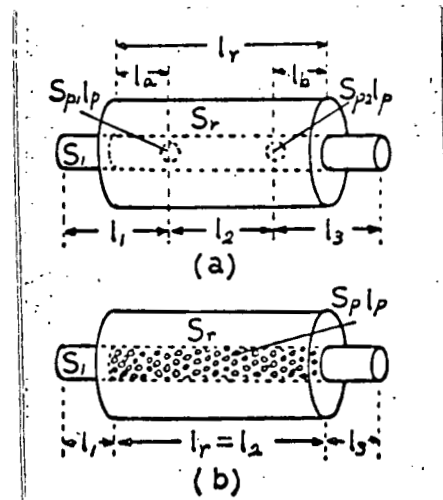


Figure 11. Example of resonator which is connected by more than two points to the pipe

$$\left. \begin{aligned} A' &= \frac{S_r}{S_1} A'' \sin kl_2 + B'' \cos kl_2, \\ D' &= \frac{S_r}{S_1} D'' \sin kl_2 + B'' \cos kl_2, \\ B' &= B'' \sin kl_2, \\ C' &= \frac{S_r}{S_1} \left\{ 2 + \left(\frac{S_1}{S_r} B'' + \frac{S_r}{S_1} C'' \right) \sin kl_2 \right. \\ &\quad \left. - (A'' + D'') \cos kl_2 \right\}, \\ E' &= \frac{S_r}{S_1} \sin kl_2 + B'' \end{aligned} \right\} \quad (28)$$

Here

$$\left. \begin{aligned} A' &= \frac{\cos k(l_2 + l_s)}{\cos kl_s} - (kl_s) \frac{S_r}{S_{r1}} C' \\ D' &= \frac{\cos k(l_2 + l_s)}{\cos kl_s} - (kl_s) \frac{S_r}{S_{r1}} C' \\ B' &= \sin kl_2 + (kl_s) \left\{ \frac{S_r \cos k(l_2 + l_s)}{S_{r1} \cos kl_s} \right. \\ &\quad \left. + \frac{S_r \cos k(l_2 + l_s)}{S_{r1} \cos kl_s} \right\} - (kl_s)^2 \frac{S_r}{S_{r1}} \frac{S_r}{S_{r1}} C' \\ C' &= \frac{\sin kl_r}{\cos kl_s \cos kl_s} \end{aligned} \right\} \quad (29)$$

In cases when the connections are distributed as shown in Figure 11(b), if we assume that kl_p is small and that the sound waves are plane waves advancing in the axial direction, and if the series impedance per unit of length in the pipe part is Z_1 , the parallel admittance is Y_1 , the sound pressure at point x in the axial direction is p_1 , the volume velocity is U_1 , and the corresponding values for the cavity part are Z_2 , Y_2 , p_2 , and U_2 , respectively, and if the connection admittance per unit of length is Y_3 , the following will apply:

$$\left. \begin{aligned} \frac{\partial U_1}{\partial x} &= -Y_1 p_1 - Y_3(p_1 - p_2), \quad \frac{\partial p_1}{\partial x} = -Z_1 U_1 \\ \frac{\partial U_2}{\partial x} &= -Y_2 p_2 + Y_3(p_1 - p_2), \quad \frac{\partial p_2}{\partial x} = -Z_2 U_2 \end{aligned} \right\} \quad (30)$$

Therefore, p_1 , p_2 , U_1 and U_2 can be found from these. If we add the condition that $U_2 = 0$ at both ends of the cavity (see Appendix 1), we obtain the following:

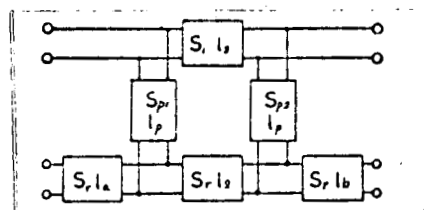


Figure 12. Equivalent network of the resonator which is illustrated in Figure 11(a). S : cross sectional area, l : effective length

$$\left. \begin{aligned}
 A' &= D' = \frac{S_r}{S_1} \sin kl_2 \cosh bkl_2 \\
 &\quad - b \cos kl_2 \sinh bkl_2 \\
 B' &= \frac{S_r}{S_1 + S_r} \left\{ 2 + \left(\frac{S_1}{S_r} \frac{1}{b} - \frac{S_r}{S_1} b \right) \right. \\
 &\quad \left. \times \sin kl_2 \sinh bkl_2 - 2 \cos kl_2 \cosh bkl_2 \right\} \\
 C' &= -\frac{S_1 + S_r}{S_1} b \sin kl_2 \sinh bkl_2 \\
 E' &= \frac{S_r}{S_1} \sin kl_2 - b \sinh bkl_2
 \end{aligned} \right\} \quad (31)$$

Here, S_p is the total connection area, and

$$b = \sqrt{\left(\frac{S_r + S_p}{S_1 + S_r} \right) \frac{1}{(kl_r)(kl_2)} - 1} \quad (32)$$

4.2. Discussion

The equations given above are quite complicated, and the numerical calculations require considerable time even if an electronic computer is used. However, the general situation is more or less the following:

1) When l_2 is extremely small, the muffler will be of the ordinary resonator type described in section 3. However, when l_2 has a value equal or close to l_r , the cavity will become extremely flat; this is not realistic. When there are two connections and one has a connecting area much smaller than the other, it is sufficient to consider only the larger one.

2) When S_r/S_1 is not much larger than 1 or smaller than 1, it is not possible to expect a very good muffler effect.

3) When $S_r/S_1 \gg 1$, kl_2 will be great, and it is almost impossible to expect any effect at all within the range where

$$kl_2 > \frac{l_2 S_p + S_r}{l_p S_1} \quad (\text{Fig. 11 (a)}) \quad (33)$$

$$kl_2 > \sqrt{\frac{l_2 (S_r + S_p)}{l_p (S_1 + S_r)}} = \sqrt{\frac{l_2 S_r}{l_p S_1}} \quad (\text{Fig. 11 (b)}) \quad (34)$$

On the contrary, within the range of small values of kl_2 , the properties will approach those of an expansion chamber type muffler. In Figure 11 (a), within the range where

$$kl_2 \ll \frac{l_2 S_p}{l_p S_1}, \quad kl_2 \ll \frac{l_2 S_r}{l_p S_1} \quad (35)$$

Equation (7) will be the following (see Appendix 2):

$$L_r \approx 20 \log \left| \frac{S_r^2 \frac{\sin kl_1 \sin kl_r \sin kl_3}{\cos kl_r \cos kl_3}}{(S_1 + S_r) S_1} \right| \quad (36)$$

In Figure 11 (b), when $l_2 S_p / l_p S_1 \gg 1$, within the range where

$$kl_2 \ll \sqrt{\frac{l_2 S_r}{l_p S_1}} \quad (37)$$

$$kl_2 \ll \frac{S_1}{S_r} \sqrt{\frac{l_2 S_r}{l_p S_1}} \sinh \sqrt{\frac{l_2 S_r}{l_p S_1}} \quad (38)$$

Equation (7) will be the following (see Appendix 2):

$$L_r \approx 20 \log \left| \frac{S_1 + S_r}{S_1} \frac{\sin kl_1 \sin kl_2 \sin kl_3}{\cos kl_2 \cos kl_3} \right| \quad (39)$$

Thus, the equation will assume the same pattern as that for an expansion chamber type muffler [1, 6]. That is, one need only consider an equivalent expansion chamber type muffler as in Figure 13. For usual dimensions, with relatively large values of l_2 and of the connection area, this range is quite extensive.

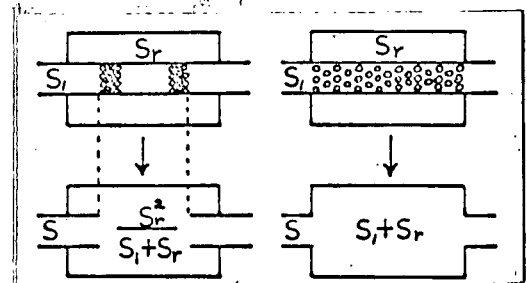


Figure 13. Equivalent expansion chamber type muffler

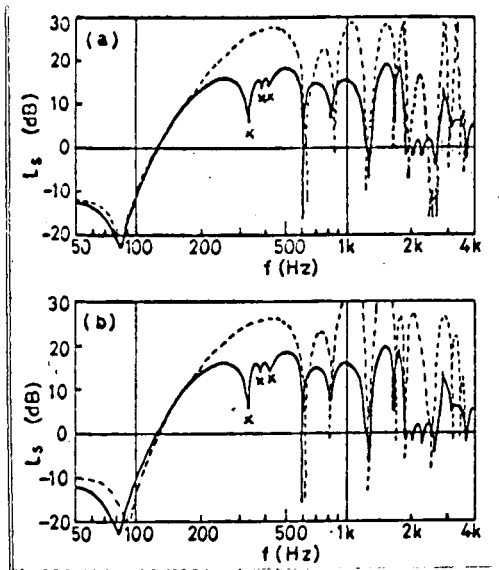


Figure 14. L_s of a muffler illustrated in Figure 11(a), (constant pressure source). The solid curves show the results of measuring and the dotted curves show those of calculation, (a): by Equation (7) and (28), (b): equivalent expansion chamber type muffler. The dips marked by x are caused by the harmonics of source.
 $l_r = 200$, $l_a = l_b = 60$, S_r : $135 \phi - 33 \phi$, $l_p = 7$, $S_{p1} = S_{p2}$: $10 \phi \times 12$, $l_1 = 135$, $l_2 = 80$, $l_3 = 266$, S_1 : 27ϕ (mm). (11°C)
 $\sqrt{l_2 S_{p1} / l_p S_1} = 21.5$, $f = 670 \text{ Hz}$
for $kl_3 = 1$

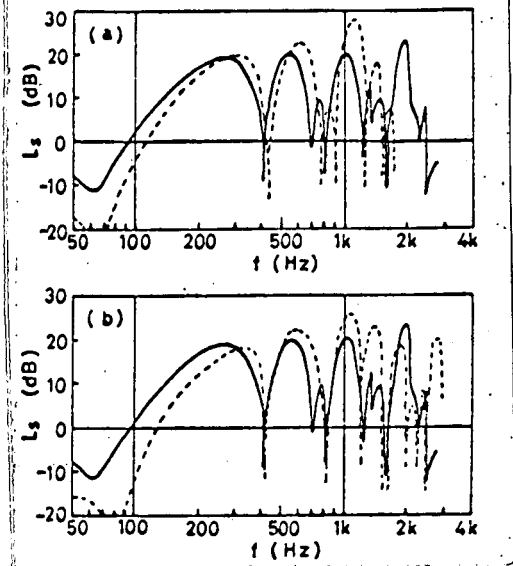


Figure 15. L_s of a muffler illustrated in Figure 11(b), (constant pressure source). The solid curves show the results of measuring and the dotted curves show those of calculation, (a): by Equation (7) and (31), (b): equivalent expansion chamber type muffler. $l_2 = 400$, S_r : $135 \phi - 33 \phi$, $l_p = 7$, S_p : $10 \phi \times 60$, $l_1 = 75$, $l_3 = 201$, S_1 : 27ϕ (mm), (11°C). $(l_2 S_p / l_p S_1)^{1/2} = 21.7$, $f = 134 \text{ Hz}$ for $kl_2 = 1$

When the E' in Equations (28) and (31) has a value of zero, resonance will occur and the properties will be similar to those of a resonator type muffler. However, a large effect can be obtained in cases when resonance occurs at very low frequencies, as was described in section 3. This occurs in cases when there

are small values of l_2 and of the connection area, and when there is almost no range in which the muffler can be considered to be an expansion chamber type muffler.

Examples of comparisons of the calculated and measured values are shown in Figures 14 and 15. Figure 16 shows the calculated values in cases when the muffler operates in the manner of a resonator type muffler. However, from the dimensional point of view this is a special case.

5. Conclusion

The general equation for the muffler properties, when a resonator type muffler is installed, was given, using the four terminal constants of an equivalent circuit.

In cases when the cavity is connected at one point with the pipe, if one ignores the loss, the properties can be found easily by plotting the construction on graph paper. It was found that there was a good effect within the range in which the resonator is considered to be a concentrated constant.

However, it is impossible to ignore the loss at the neck part of the resonator, and it is difficult to establish this

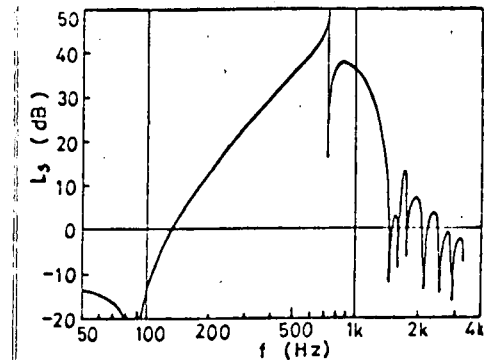


Figure 16. Theoretical L_s of a muffler illustrated in Figure 11(b), (constant pressure source). $l_2 = 100$, $S_r: 200 \phi - 33 \phi$, $l_p = 7$, $S_p: 5 \phi \times 30$, $l_1 = 75$, $l_3 = 206$, $S_1: 27 \phi$ (mm), (11°C). $a = (l_2 S_p / l_p S_1)^{1/2} = 3.83$, $(S_1 S_r) \times a \times (\sinh a) = 1.66$. $f = 537 \text{ Hz}$ for $kl_2 = 1$.

/371

loss accurately. Therefore, if we assume the loss in a manner which makes it convenient to plot the graph, it becomes extremely easy to plot the graph. Furthermore, it was found to be possible to represent the properties sufficiently for all practical purposes.

We further determined the theoretical equations for cases where the resonator was connected to the pipe in two places, or when the connections were distributed. It was found that, in the usual type where the cavity and connection areas are quite large, the muffler may be considered to be an expansion chamber type resonator within the low frequency range. There is very little sense in having connections in two places, but if the connections are distributed it will no doubt be possible to expect an effect resulting from the loss at these parts.

In resonator type mufflers, it is known that the effects of the air flow are especially great. In this case, in cases where there is no air flow, we assumed the loss in a manner convenient for plotting the graph, and an approximation was made which is effective for all practical purposes. However, it will be necessary to make further studies on the loss in the neck section of the resonator, including cases when there is air flow.

Calculations on the basis of Equations (28) and (31) were carried out at the computer room at Kumamoto University. The author thanks Messrs. Yoshida Shun'ichi (now employed at Mitsubishi Electric Corporation) and specialist Nishimura Yoshitaka of the research laboratory, as well as the personnel of the computer room, for their cooperation in the research.

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APPENDICES

1. Derivation of Equation (31)

From Equation (30):

$$\left. \begin{aligned} \frac{d^2 p_1}{dx^2} &= (Z_1 Y_1 + Z_1 Y_2) p_1 - Z_1 Y_2 p_2 \\ \frac{d^2 p_2}{dx^2} &= (Z_2 Y_1 + Z_2 Y_2) p_2 - Z_2 Y_1 p_1 \end{aligned} \right\} \quad (A)$$

If the propagation constants here are γ_1 and γ_2 , we can assume that

$$\left. \begin{aligned} p_1 &= A_1' e^{\gamma_1 x} + K_1 p_2 \\ p_2 &= A_2' e^{\gamma_2 x} + K_2 p_1 \end{aligned} \right\} \quad (B)$$

Differentiating twice with respect to x , we can rewrite this and find γ_1 , γ_2 , K_1 , and K_2 , comparing with Equation (A):

$$\left. \begin{aligned}
 \gamma_1 &= \pm \sqrt{Z_2 Y_2 + Z_2 Y_3 + Z_1 Y_3 / K_1} \\
 \gamma_2 &= \pm \sqrt{Z_1 Y_1 + Z_1 Y_3 - Z_1 Y_2 / K_1} \\
 K_1 &= \{ -(Z_1 Y_1 + Z_1 Y_3 - Z_2 Y_2 - Z_2 Y_3) \\
 &\quad \pm \sqrt{(Z_1 Y_1 + Z_1 Y_3 - Z_2 Y_2 - Z_2 Y_3)^2 \\
 &\quad + 4 Z_1 Z_2 Y_3^2} \} \frac{1}{2 Z_2 Y_2} \\
 K_2 &= -K_1 \frac{Z_2}{Z_1}
 \end{aligned} \right\} \quad (C)$$

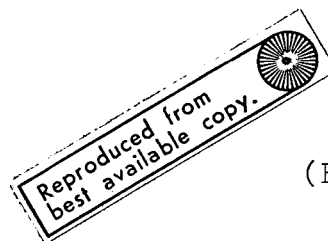
If we take the positive signs for γ_1 , γ_2 , and K_1 , Equation (B) will become:

$$\left. \begin{aligned}
 p_1 &= A_1' e^{\gamma_1 x} + B_1' e^{-\gamma_1 x} + K_1 p_2 \\
 &= A_1 \cosh \gamma_1 x + B_1 \sinh \gamma_1 x + K_1 p_2 \\
 p_2 &= A_2' e^{\gamma_2 x} + B_2' e^{-\gamma_2 x} + K_2 p_1 \\
 &= A_2 \cosh \gamma_2 x + B_2 \sinh \gamma_2 x + K_2 p_1
 \end{aligned} \right\} \quad (D)$$

U_1 and U_2 can be obtained by substituting Equation (D) into Equation (30).

If there is no loss:

$$\left. \begin{aligned}
 Z_1 &= j\omega \frac{\rho}{S_1}, \quad Z_2 = j\omega \frac{\rho}{S_r} \\
 Y_1 &= j\omega \frac{S_1}{\rho c^2}, \quad Y_2 = j\omega \frac{S_r}{\rho c^2}, \quad Y_3 = \frac{S_r}{j\omega \rho l_1 l_2}
 \end{aligned} \right\} \quad (E)$$



From this,

$$\sqrt{Z_1 Y_1} = \sqrt{Z_2 Y_2} = jk = j\frac{\omega}{c} \quad (F)$$

are each the propagation constants in each case when they are alone, and

$$\left. \begin{aligned}
 K_1 &= 1, \quad K_2 = -\frac{S_1}{S_r} \\
 \gamma_1 &= jk \sqrt{\left(\frac{S_r}{S_1} + \frac{S_r}{S_r}\right) \frac{1}{(kl_1)(kl_2)} - 1} = jbk \\
 \gamma_2 &= jk
 \end{aligned} \right\} \quad (G)$$

or

$$\sqrt{\frac{Z_1}{Y_1}} = \frac{\rho c}{S_1}, \quad \sqrt{\frac{Z_2}{Y_2}} = \frac{\rho c}{S_r} \quad (H)$$

are the characteristic impedances in each case when they are alone.

From the conditions where $U_2 = 0$ at $x = 0$ and $x = l_2$, we can establish A_1, A_2, B_1 , and B_2 and seek the relationships between p_1 and U_1 when $x = 0$ and p_1 and U_1 when $x = l_2$. In this case, the four terminal constants are sought in the same manner as in Equation (31).

2. Derivation of Equations (36) and (39)

From Equation (35) it is evident that

$$\left[\frac{S_r}{S_1} \gg kl, \frac{S_r}{S_n}, \frac{S_r}{S_1} \gg kl, \frac{S_r}{S_n} \right] \quad (I)$$

Besides, since $S_r/S_1 \gg 1$, with Equations (28) and (29) substituted in Equation (7), we can obtain Equation (36), if we ignore the terms except those related to S_r/S_1 .

In view of the fact that $S_r/S_1 \gg 1$ and in view of Equation (37), Equation (32) will be

$$b \approx \frac{1}{kl_2} \sqrt{\frac{l_2 S_r}{l_1 S_1}} \gg 1 \quad (J)$$

Since $\sqrt{l_2 S_p / l_p S_1}$ has a value considerably larger than 1,

$$\sinh bkl_2 \approx \cosh bkl_2 \gg 1 \quad (K)$$

In view of Equation (J) and Equation (38),

$$b \sinh bkl_2 \gg \frac{S_r}{S_1} \quad (L)$$

In Equation (31), if we ignore the terms except those related to $\sinh bkl_2$, $\cosh bkl_2$, and $b \sinh bkl_2$, and substitute these into Equation (7), ignoring the terms except those related to S_r/S_1 , and further ignoring the terms related to $1/b$, we will obtain Equation (39).

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